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On the Development of Fracture Models for Metal Forming

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Abstract—An increase in the ductility of cast metals due to working of their structures by preliminary deformation under pressure is taken into consideration within the framework of a phenomenological model of fracture and a model based on the theory of an inhomogeneous porous medium.

INTRODUCTION

It is known that metal forming of brittle metals can improve their plastic properties [1, 2]. An increase in the ductility of metals whose brittleness is caused by the presence of a large number of micropores and microcracks is due to healing of these defects upon plastic deformation under pressure. This effect was first discovered by Beresnev's group [2] during the experimental study of deformation of brittle copper. The theoretical description of an increase in ductility associated with the strain healing of microdiscontinuities is provided by the Kolmogorov model in its original form [3] and fracture models based on the theory of an inhomogeneous porous medium [4–6].

The brittleness of cast metals has a different physical nature, namely, dendrite and coarse-grained structures and brittle phases precipitating along grain boundaries during solidification. The latter variant is typical, for example, of secondary nonferrous metals, which are characterized by a large amount of almost irremovable impurities that form intermetallic compounds [7]. If this is the case, an increased ductility of metals after metal forming can be attributed to the transformation of a cast structure into a deformed structure induced by the disintegration and dissolution of brittle inclusions. Such a process is often referred to as working of a metal structure [8].

The effects of increasing ductility after working of a metal structure still needs to be accounted for by the models of fracture upon metal forming.

In this study, we propose possible ways of taking these effects into consideration when describing the fracture of metals within the framework of phenomenological models [3, 9, 10] and a fracture model based on the theory of an inhomogeneous porous medium [4–6].

PHENOMENOLOGICAL MODELS

Phenomenological models of fracture during metal forming postulate the existence of a macroscopic quantity (damage ω) that is a quantitative measure of microfracture during deformation. Macroscopic fracture occurs when the damage reaches a certain critical value ω_p. The relative damage $\psi = \omega/\omega_p$ is called the expenditure of the plasticity reserve. Thus, the criterion of fracture has the form $\psi = 1$.

It is usually assumed that an increment $d\psi$ caused by deformation is proportional to an increment in the shear strain $d\Lambda$:

$$d\psi = d\Lambda/\Lambda_p. \qquad (1)$$

The value of Λ_p is a function of the rigidity index of the stressed state $\eta = \sigma/T$ (here, σ is the hydrostatic stress and T is the shear stress), the Lode coefficient μ_σ, the shear strain rate H, and the temperature θ.

According to Eq. (1), the rate of microdamage accumulation in a material upon plastic deformation is proportional to Λ_p^{-1}.

With reference to (1), the expenditure of the plasticity reserve is defined by the ratio

$$\psi = \int_0^{\Lambda} \frac{d\Lambda}{\Lambda_p},$$

and the fracture criterion has the form

$$\int_0^{\Lambda_c} \frac{d\Lambda}{\Lambda_p} = 1,$$

where Λ_c is the critical shear strain corresponding to the onset of macroscopic fracture.

The latter expression reveals one more physical meaning of Λ_p as the shear strain that is accumulated in a material until fracture under the conditions when the thermomechanical parameters that govern Λ_p are maintained constant throughout the loading procedure.

Let us take into consideration the fact that the rate of microdamage accumulation depends on the structure of brittle phases. For this purpose, we rewrite Eq. (1) with the use of a parameter ξ, which accounts for the effect of structure on fracture, and, thus, obtain the following equation for the increment ψ:

$$d\psi = \xi d\Lambda/\Lambda_p^*, \tag{2}$$

where Λ_p^* is a function of the same parameters as Λ_p (it will be defined below). Note that our model implies $\Lambda_p^* \neq \Lambda_p$, where the right-hand side of the inequality still has the meaning of the shear strain to fracture at constant thermomechanical parameters of the loading process.

It is evident that ξ depends on the history of deformation. To a first approximation, we assume that ξ is merely specified by the shear strain:

$$\xi(\Lambda) = 1 + A\exp(-\beta\Lambda), \tag{3}$$

where the parameters A and β characterize the intensity of transformations of a structure under plastic deformation. They are naturally governed by the nature of a metal and the characteristics of its state of stress. In this study, where our aim is only to establish the possibility of taking into consideration the effects of metal working, these parameters are taken to be constant.

Combining Eqs. (2) and (3) shows that the rate of microdamage accumulation decreases with increasing deformation as a result of metal working. Using Eqs. (2) and (3), the expenditure of the plasticity reserve can be expressed as

$$\psi = \int_0^{\Lambda} \frac{[1 + A\exp(-\beta\Lambda)]d\Lambda}{\Lambda_p^*}, \tag{4}$$

and the fracture criterion takes the form

$$\int_0^{\Lambda_c} \frac{[1 + A\exp(-\beta\Lambda)]d\Lambda}{\Lambda_p^*} = 1. \tag{5}$$

In the case when the thermomechanical parameters are kept constant during loading, $\Lambda_c = \Lambda_p$, Eq. (5) transforms into the expression

$$\int_0^{\Lambda_p} [1 + A\exp(-\beta\Lambda)]d\Lambda = \Lambda_p^*. \tag{6}$$

Integration of Eq. (6) yields

$$\Lambda_p^* = \Lambda_p + \frac{A}{\beta}[1 - \exp(-\beta\Lambda_p)],$$

and substitution off this expression into the formula for relative damage (4) gives

$$\psi = \int_0^{\Lambda} \frac{[1 + A\exp(-\beta\Lambda)]d\Lambda}{\Lambda_p + A\beta^{-1}[1 - \exp(-\beta\Lambda_p)]}.$$

Let a cast metal sample with a rigidity index $\eta_1 < 0$ be preliminarily deformed to a shear strain Λ_1. In this way, a sample with a worked structure is fabricated. The ductility Λ_{p1} of this metal at constant values of thermomechanical parameters can be determined from the equation

$$\int_0^{\Lambda_{p1}} \frac{\{1 + A\exp[-\beta(\Lambda + \Lambda_1)]\}d\Lambda}{\Lambda_p(\eta) + A\beta^{-1}\{1 - \exp[-\beta\Lambda_p(\eta)]\}} = 1 - k\psi_1,$$

where ψ_1 is the expenditure of the plasticity reserve within preliminary deformation,

$$\psi_1 = \int_0^{\Lambda_1} \frac{\{1 + A\exp(-\beta\Lambda)\}d\Lambda}{\Lambda_p(\eta)_1 + A\beta^{-1}\{1 - \exp[-\beta\Lambda_p(\eta_1)]\}};$$

and k is the coefficient taking into account possible recovery of the ductility during a heat treatment after the preliminary deformation.

For the sake of comparison, Fig. 1 shows the plasticity curves of the as-cast ($\Lambda_p(\eta)$) and predeformed ($\Lambda_{p1}(\eta)$) metals. For the calculation, the function $\lambda_p(\eta)$ is taken in the form

$$\Lambda_p(\eta) = \frac{\Lambda_{pk}}{1 + 3^{1/2}\eta}\exp(-3^{1/2}\eta),$$

which is known [11] to provide a close fit to the experimental plasticity curves of metals that are brittle upon tension and highly resistant to compression. Cast lithium presents an example of such behavior. Numerical estimations are carried out for the following values of parameters: $\Lambda_{pk} = 0.1$, $A = 10^2$, $\beta = 10$, $\eta_1 = -0.55$, $\lambda_1 = 1$, and $k = 1$.

The results obtained indicate that the proposed model accounts for the increase in the ductility of the as-cast metal after working its structure.

Figure 2 clearly illustrates an enhanced ductility of cast zinc after deformation by hydroextrusion. The as-cast sample displays brittle fracture, whereas the predeformed zinc can easily be bent without failure.

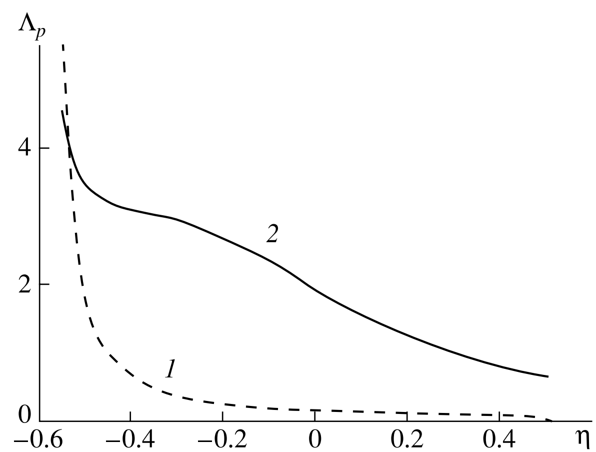

Fig. 1. Plasticity curves for (*1*) as-cast and (*2*) predeformed metal.

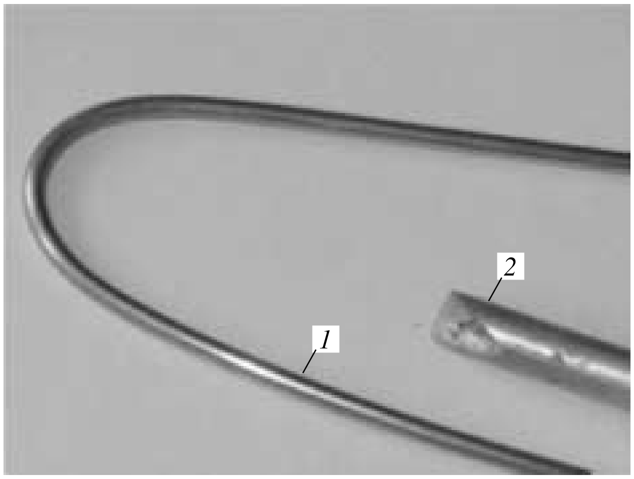

Fig. 2. Samples of (*1*) as-cast and (*2*) predeformed zinc.

FRACTURE MODEL BASED ON THE THEORY OF AN INHOMOGENEOUS POROUS MEDIUM

The model is based on the following statements:

(1) plastic deformation of metals is accompanied by both the appearance and healing of micropores;

(2) as a result of micropore accumulation, the ductility of metal reduces and macroscopic fracture can occur when the total volume of micropores per unit volume (i.e., porosity) reaches a certain critical value; and

(3) the critical porosity of many metals is close to ~0.01 and is nearly independent of a deformation scheme [12].

The dependence of the porosity on the rigidity index of a state of stress and the deformation of a metal can be found from the kinetic equation [4–6]

$$d\theta/d\gamma = \alpha + 6a\theta\eta, \qquad (7)$$

where θ is the porosity, $\gamma = \Lambda/2^{1/2}$, α is the accommodation parameter characterizing the rate of micropore formation, and a is the parameter of the pore shape.

The first term in the right-hand side of Eq. (1) describes micropore nucleation, and the second term describes their healing at $\eta < 0$, i.e., at compressive stresses.

All of known mechanisms of pore formation [6] can eventually be reduced to the deformation-induced accommodation of structural elements of a material. For example, in a deformed material with an undeformable inclusion, a pore can form at the boundary of the inclusion with the surrounding material due to the fact that the flowing material is always in full contact with the inclusion.

The accommodation parameter α characterizes the micropore nucleation rate during plastic deformation;i.e., it presents a quantitative measure of the ability of structural elements to accommodate. Complete accommodation in materials corresponds to $\alpha = 0$. The value of α grows with the number of obstacles to cooperative plastic deformation; i.e., the less efficient the accommodation mechanisms, the higher the value of α.

Let us now rewrite Eq. (7) in such a way as to take into account the specific features of deformation of cast metals caused by brittle layers along grain boundaries. To this end, the parameter α is presented as the sum

$$\alpha = C_1\alpha_1 + C_2\alpha_2, \qquad (8)$$

where the first and the second terms describe micropore formation along grain boundaries due to failure of the brittle layers and micropore formation in the bulk of grains, respectively; α_1 and α_2 are the rates of pore formation because of the failure of grain boundaries and due to the other causes (which are considered in detail in [6]), respectively; and C_1 and C_2 are the relative contributions of these mechanisms to the total process of pore formation, i.e.

$$C_1 + C_2 = 1. \qquad (9)$$

As the deformation grows, C_1 should decrease because of the fracture of the brittle layers. We will search for the dependence of C_1 on γ under the natural assumption that a decrease in the volume of the brittle interlayers during deformation is proportional to this volume and the deformation increment. Hence, it follows that

$$dC_1 = -\lambda C_1 d\gamma, \qquad (10)$$

where λ is a proportionality factor.

Equation (10) should be integrated with the initial condition $C_1 = 1$ at $\gamma = 0$, which follows from the fact that, at the initial stage of deformation, the microfracture of a material is mainly conditioned by the presence of the brittle intergrain layers.

Integrating Eq. (10) gives C_1 in the form

$$C_1 = \exp(-\lambda\gamma). \qquad (11)$$

From this expression, the physical meaning of the parameter λ is that $1/\lambda$ is equal to the deformation at which the volume of the interlayer skeleton decreases by a factor of *e*.

Substituting Eq. (11) into (8) and taking into consideration Eq. (9), we obtain

$$\alpha = \alpha_1\exp(-\lambda\gamma) + \alpha_2[1 - \exp(-\lambda\gamma)]. \qquad (12)$$

The parameter α_1 characterizes micropore formation due to intergranular fracture of an alloy due to the brittle interlayers. In terms of the approach accepted, the structure of such an alloy is similar to that of a powder material, where the bonds between particles can easily be broken under the action of tensile stresses. Therefore, the parameter α_1 should be of the same order of magnitude as the accommodation parameters of powder materials. According to the data in [6], $\alpha_1 \approx$ 0.1–1.0.

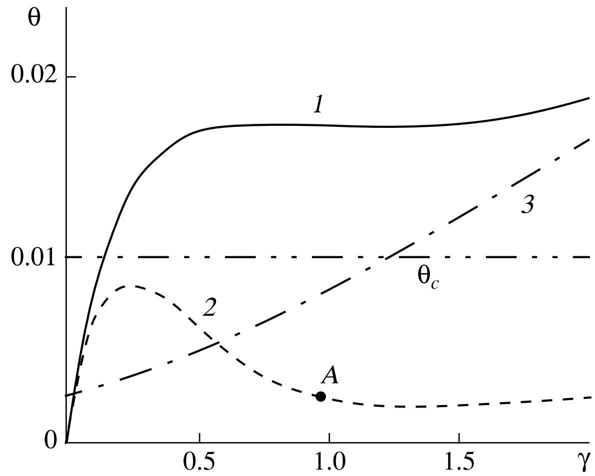

Fig. 3. Deformation-induced porosity θ vs. the degree of deformation γ for (*1*, *2*) as-cast and (*3*) predeformed metal with the rigidity indices $\eta = -3^{-1/2}$ (*1*, *3*) and –4 (*2*). θ is the critical value of porosity.

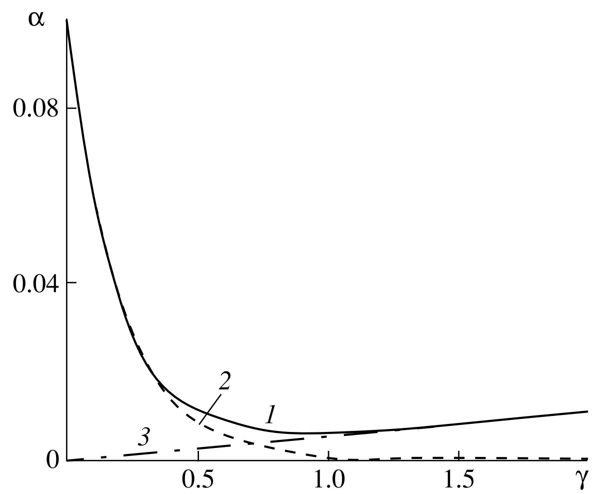

Fig. 4. Accommodation parameters (*1*) α, (*2*) α_1, and (*3*) α_2 vs. the degree of deformation.

The parameter α_2 is related to the mechanisms of pore formation in the bulk of grains. According to the data in [6], in metals, $\alpha_2 \approx 10^{-3}$–10^{-2}, and the parameter has a power dependence on strain:

$$\alpha_2 = \alpha_2' \gamma^n, \tag{13}$$

where α_2' is a constant and n is a parameter (of an order of ≈0.1–1.0).

After the substitution of Eq. (13) into (12), we have

$$\alpha = \alpha_1 \exp(-\lambda\gamma) + \alpha_2'[1 - \exp(-\lambda\gamma)]\gamma^n.$$

We use this expression to replace the accommodation parameter in Eq. (7) and to arrive finally at a kinetic equation for the deformation-induced porosity of materials with brittle intergrain layers in the as-cast state:

$$d\theta/d\gamma = \alpha_1 \exp(-\lambda\gamma) + \alpha_2'[1 - \exp(-\lambda\gamma)]\gamma^n + 6a\theta\eta. \tag{14}$$

In order to study the dependence of θ on γ, we numerically solve Eq. (14) with the following typical values of parameters: $\alpha_1 = 0.1$, $\alpha_2' = 0.005$, $a = 0.1$, $\lambda = 5$, $n = 1$, and $\theta_0 = 0$.

Figure 3 shows the dependences θ = θ(γ) at two stressed states with different rigidity indices. The region of material fracture lies above the critical value of porosity, which is taken to be 0.01 [12]. It is seen that, upon upsetting at $\eta = -3^{-1/2}$, a material with a brittle intergrain skeleton (as-cast sate) fails even at a deformation of γ ≈ 0.2. A high-pressure (η = –4) treatment of this material not only allows this material to withstand high deformations without fracture but also considerably reduces the accommodation parameter (Fig. 4). The latter circumstance signifies that this material can be deformed without fracture at a lower level of hydrostatic pressure.

In order to illustrate this conclusion, we consider a material with parameters corresponding to, e.g., point *A* in Fig. 3 and calculate a change in its porosity upon upsetting ($\eta = -3^{-1/2}$) using Eq. (14). The results obtained (Fig. 3) indicate that this material can be deformed by upsetting without fracture to γ ≈ 1.3, which is appreciably higher than the value typical of as-cast metals (γ ≈ 0.2).

Analysis of kinetic equation (14) demonstrates that it offers a proper qualitative description of microfailure in materials with a brittle intergrain skeleton and takes into account the basic feature of these materials: an increase in the ductility upon high-pressure deformation.

Some quantitative estimations can be performed after experimental determination of the parameters α_1, α_2', a, λ, and n. This will be done in our next works. Once the models proposed are checked experimentally, they can be used for predicting the technological ductility of cast metals and for designing optimal schemes of their deformation.

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