Modular Learning

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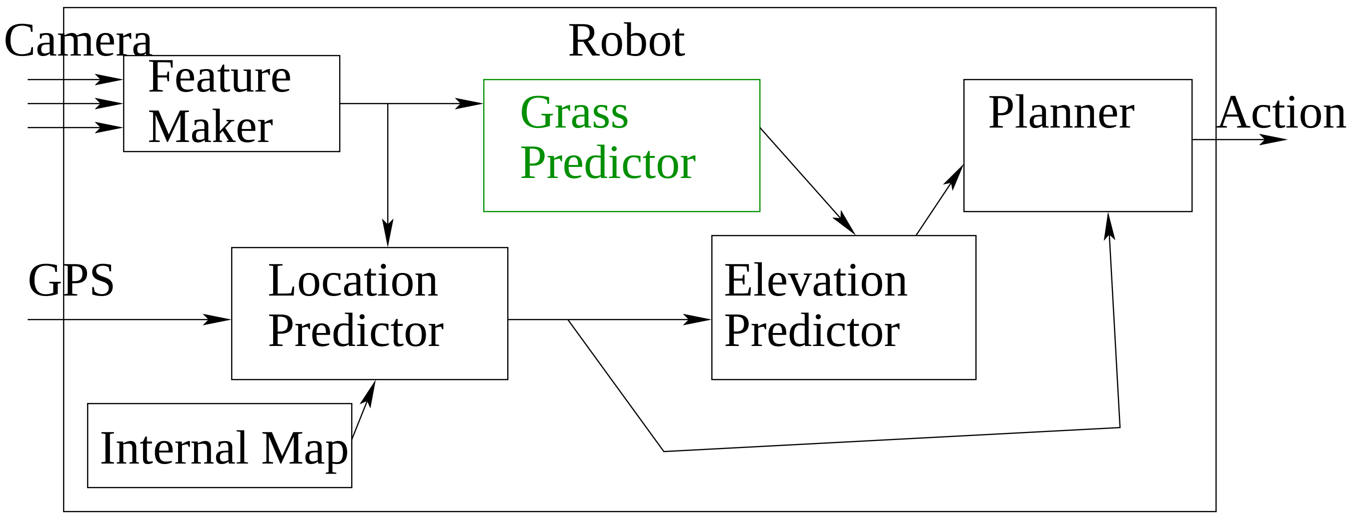
For COMS-4771

Real Learning Systems are complicated



How do we learn the parameters of the grass predictor?

Outline

1. Subproblem Learning

2. End-to-End Learning

3. Extensions of Gradient Descent

Subproblem Learning is Powerful

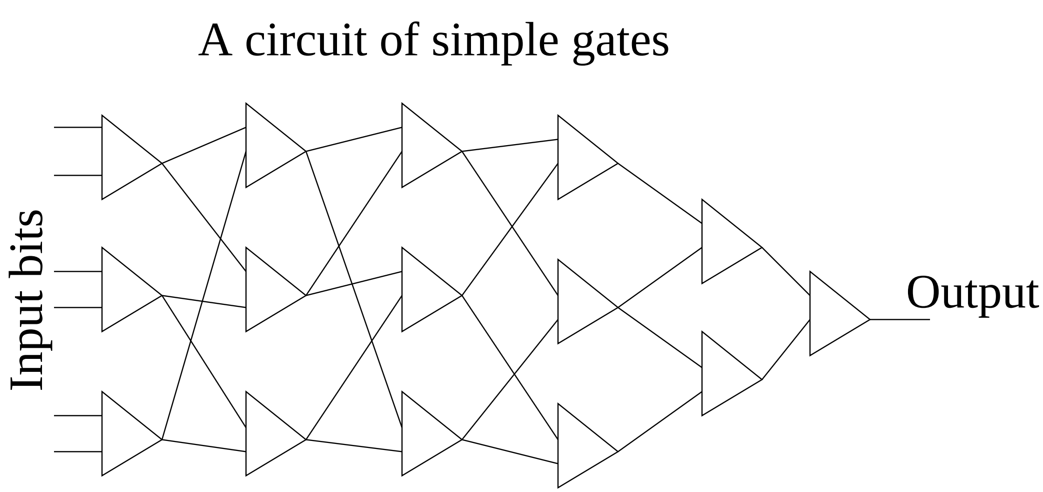
Theorem: Assuming AES encryption is unbreakable, there exists learning problems D for which direct learning of subproblems $D_1, ..., D_m$ is tractable, yet learning without subproblems is computationally intractable.

In other words: learning the full problem can be hard, but if you know the right subproblems to solve, it can become easy.

Proof: Let D be a distribution on $x =$ AES encrypted IMs and $y =$ plain text IMs.

1. y is essentially unpredictable given x.

2. But AES can be written as a circuit of and/or/not gates.



and "and", "or", and "not" are all learnable.

A problem with Subproblem Learning

Theorem: (Independent Learning Weakness) For any m, there exists a learning problem D with subproblems $D_1, ..., D_m$ such that:

$$e(C_f, D) = \sum_i e(f_i, D_i)$$

where $e(C_f, D)$ is the error rate of the circuit C composed of the learned $f_1, ..., f_m$.

Proof: Create a circuit where any error at any gate implies an overall error.

Implication: erring on any subproblem can cause an overall error.

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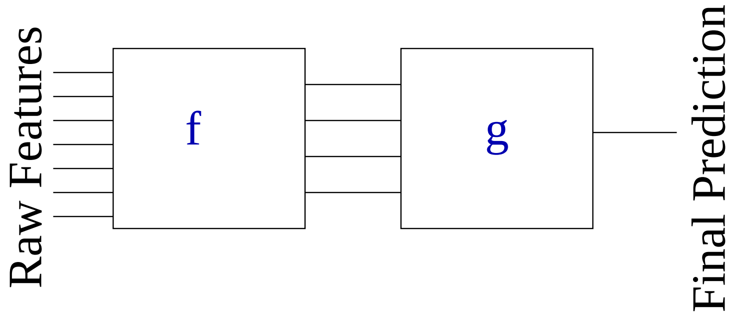
End-to-End learning

Essential idea: do a joint optimization of all subproblems to improve performance.

Primary method: gradient descent

A Simplification of the Problem



Function to learn = $g(w_g, f(w_f, x))$

Suppose we care about squared loss: $E_{x,y \sim D}(g(w_g, f(w_f, x)) - y)^2$

How should we tune w_g and w_f?

Gradient Descent

$$-\frac{\partial}{\partial w_g}(g(w_g, f(w_f, x)) - y)^2$$

$$= 2(g(w_g, f(w_f, x)) - y)\frac{\partial g(w_g, f(w_f, x))}{\partial w_g}$$

and

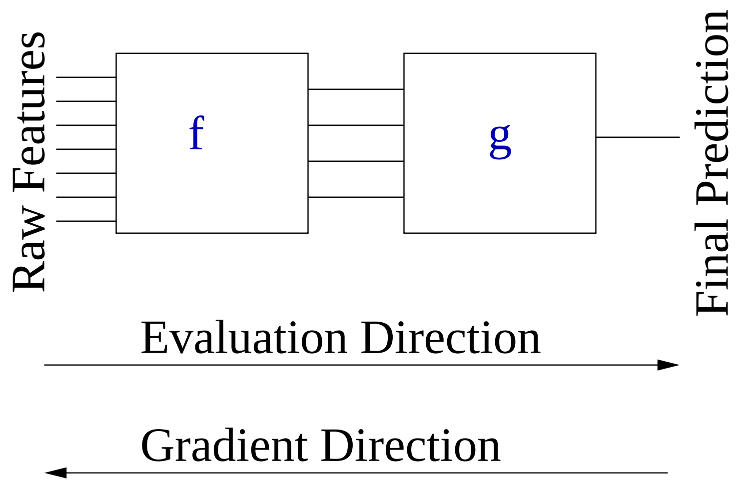
$$-\frac{\partial}{\partial w_f}(g(w_g, f(w_f, x)) - y)^2$$

$$= 2(g(w_g, f(w_f, x)) - y)\frac{\partial g(w_g, f(w_f, x))}{\partial w_f}$$

$$= 2(g(w_g, f(w_f, x)) - y)\frac{\partial g(w_g, f(w_f, x))}{\partial f(w_f, x)}\frac{\partial f(w_f, x)}{\partial w_f}$$

"Chain rule of differentiation"

Information Flow



Gradient chain rule goes in the opposite direction to evaluation.

Some notes about chain rule learning

- Needs continuous functions.

- In general, local minima bite, unless the function is convex.

- Sigmoid $h(x) = \frac{1}{1+e^{-x}}$ is convenient: $\frac{\partial h(x)}{\partial x} = h(x)(1 - h(x))$

- Since derivatives are linear, if g uses f twice (happens all the time in a big circuit), the updates to w_f sum.

- The update can be online.

- The derivative on w_f can collapse to zero very quickly with depth of a circuit. Most people use shallow structures (See Yann LeCun's Convolutional Neural Networks for a nonshallow network).

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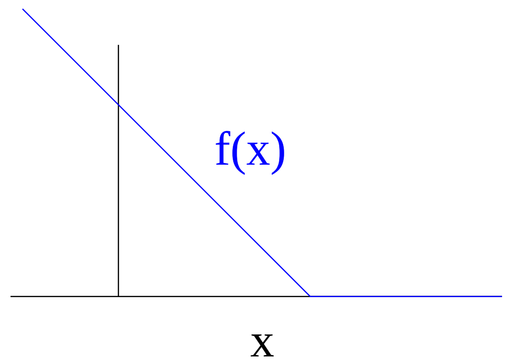
Problem: Your derivative is discontinuous



Solution: Ignore the problem—you never land on the discontinuity in practice.

(See the study of "subgradients".)

Problem: a set of weights must sum to 1

Solution:

1. compute a derivative

2. gradient descent step

3. project back into the allowed set

(See “extragradient” for more details.)

Problem: function is not differentiable at all

Solution:

1. Try computing a discrete gradient: test how small changes in input alter output. Treat the discrete gradient as a gradient.

2. Find some approximation which is differentiable.

General Strategy for coping with Modular learning problems

1. Take advantage of all subproblem knowledge you have first.

2. Apply (extra|sub|discrete)gradient for final tuning.