Exploration for Evaluation and Optimization

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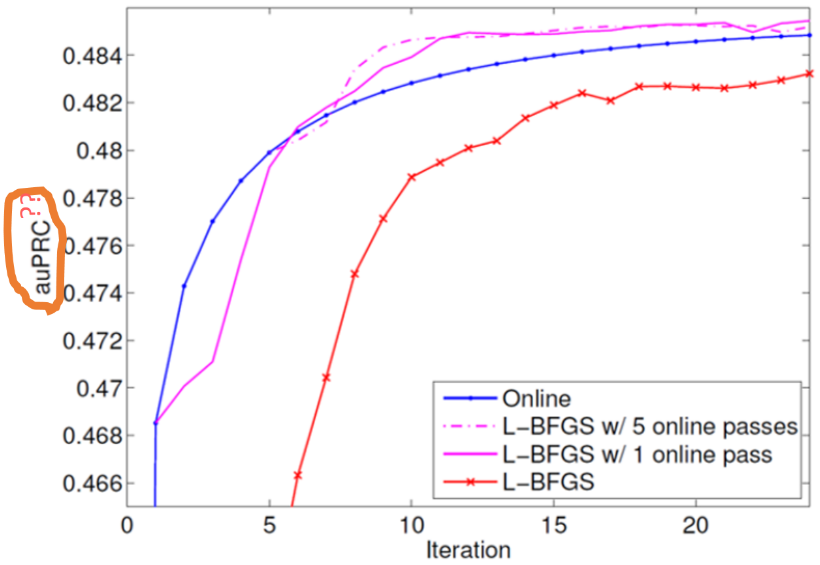
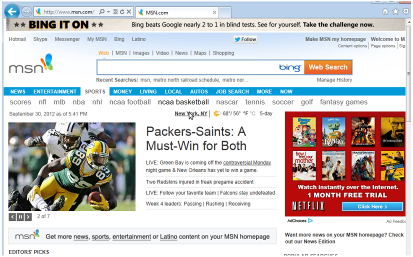
git clone git://github.com/JohnLangford/vowpal_wabbit.git



0.484
0.482
0.48
0.478
0.476
0.474
0.472
0.47
0.468
0.466
auPRC ??
0
5
10
15
20
Iteration
Online
L–BFGS w/ 5 online passes
L–BFGS w/ 1 online pass
L–BFGS

Examples of Interactive Learning



Repeatedly:

1. A user comes to Microsoft (with history of previous visits, IP address, data related to an account)
2. Microsoft chooses information to present (urls, ads, news stories)
3. The user reacts to the presented information (clicks on something, clicks, comes back and clicks again,...)

Microsoft wants to interactively choose content and use the observed feedback to improve future content choices.

Another Example: Clinical Decision Making

"I stopped taking the medicine because I prefer the original disease to the side effects."

Repeatedly:

1. A patient comes to a doctor with symptoms, medical history, test results
2. The doctor chooses a treatment
3. The patient responds to it

The doctor wants a policy for choosing targeted treatments for individual patients.

Example 3: User Interfaces

emotiv
you think, therefore, you can

The Contextual Bandit Setting

For $t = 1, \ldots, T$:

1. The world produces some context $x \in X$
2. The learner chooses an action $a \in A$
3. The world reacts with reward $r_a \in [0, 1]$

Goal: Learn a good policy for choosing actions given context.

The Evaluation Problem

Let $\pi : X \to A$ be a policy mapping features to actions. How do we evaluate it?

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Method 1: Deploy algorithm in the world.

Very Expensive!

Method 2: The "Direct method"

Use past data to learn a reward predictor $\hat{r}(x, a)$, and act according to $\arg\max_a \hat{r}(x, a)$.

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Example: Deployed policy always takes a_1 on x_1 and a_2 on x_2.

	a_1	a_2
x_1		
x_2		

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Observed

	a_1	a_2
x_1	.8	?
x_2	?	.2

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Observed/Estimated

	a_1	a_2
x_1	.8/.8	?/.5
x_2	?/.5	.2 /.2

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	a_1	a_2
x_1	.8/.8	?/.514
x_2	.3/.3	.2 /.014

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Observed/Estimated/True

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Basic observation 1: Generalization insufficient.

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Basic observation 2: Exploration required.

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Basic observation 3: Errors $\neq$ exploration.

Method 3: The Importance Weighting Trick

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One answer: Collect T exploration samples of the form

$$(x, a, r_a, p_a),$$

where
$x =$ context
$a =$ action
$r_a =$ reward for action
$p_a =$ probability of action a
then evaluate:

$$\text{Value}(\pi) = \text{ Average}\left(\frac{r_a \mathbf{1}(\pi(x) = a)}{p_a}\right)$$

The Importance Weighting Trick

Theorem

For all policies π, for all IID data distributions D, Value(π) is an unbiased estimate of the expected reward of π:

$$\mathbf{E}_{(x,\vec{r})\sim D}\left[r_{\pi(x)}\right] = \mathbf{E}[\,\text{Value}(\pi)\,]$$

with deviations bounded by

$$O\left(\frac{1}{\sqrt{T \min_x p_{\pi(x)}}}\right)$$

Proof: $\mathbf{E}_{a\sim p}\left[\frac{r_a \mathbf{1}(\pi(x)=a)}{p_a}\right]$

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Action	1	2
Reward	0.5	1
Probability	$\frac{1}{4}$	$\frac{3}{4}$
Estimate		

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$$\text{Value}'(\pi) = \text{Average}\left(\frac{(r_a - \hat{r}(a, x))\mathbf{1}(\pi(x) = a)}{p_a} + \hat{r}(\pi(x), x)\right)$$

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Let $\Delta(a,x) = \hat{r}(a,x) - E_{\vec{r}|x} r_a$ = reward deviation
Let $\delta(a,x) = 1 - \frac{p_a}{\hat{p}_a}$ = probability deviation

Theorem

For all policies π and all $(x,\vec{r})$:

$$|\text{Value}'(\pi) - E_{\vec{r}|x}[r_{\pi(x)}]| \le |\Delta(\pi(x),x)\delta(\pi(x),x)|$$

The deviations multiply, so deviations < 1 means we win!

How do you test things?

Use format:

action:cost:probability | features

Example:

1:1:0.5 | tuesday year million short compan vehicl line stat financ commit exchang plan corp subsid credit issu debt pay gold bureau prelimin refin billion telephon time draw basic relat file spokesm reut secur acquir form prospect period interview regist toront resourc barrick ontario qualif bln prospectus convertibl vinc borg arequip

...

How do you train?

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Reduce to cost-sensitive classification.

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Cost-sensitive multi-class classification

Distribution D over $X \times [0,1]^k$, where a vector in $[0,1]^k$ specifies the cost of each of the k choices.

Find a classifier $h : X \to \{1, \ldots, k\}$ minimizing the expected cost

$$\text{cost}(h, D) = \mathbf{E}_{(x,c)\sim D}[c_{h(x)}].$$

How do you train?

1. Learn $\hat{r}(a, x)$.
2. Compute for each x the double-robust estimate for each $a' \in \{1, ..., K\}$:

$$\frac{(r - \hat{r}(a, x))I(a' = a)}{p(a|x)} + \hat{r}(a', x)$$

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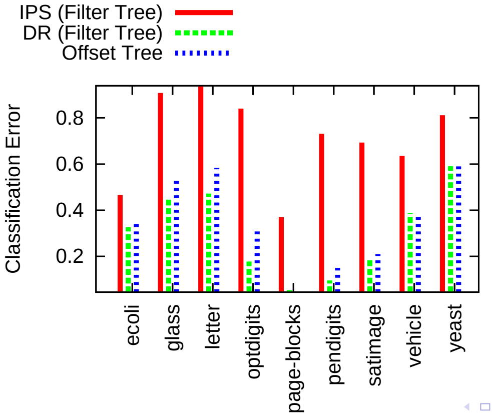
3. Learn π using a cost-sensitive classifier.

vw –cb 2 –cb_type dr rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.25
Progressive 0/1 loss: 0.0460
vw –cb 2 –cb_type ips rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.125
Progressive 0/1 loss: 0.0511
vw –cb 2 –cb_type dm rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.125
Progressive 0/1 loss: 0.0468

Experimental Results

IPS = $\hat{r}(a, x) = 0$
DR = $\hat{r}(a, x) = w_a \cdot x$
Filter Tree = Cost Sensitive Multiclass classifier
Offset Tree = Earlier method for CB learning with same representation



Train method 2: Multitask Regression

Importance Weighted Multitask Regression

Distribution D over $X \times \mathbb{R} \times \mathbb{R} \times \{1, ..., K\}$, where each task has a value with an importance weight.

Find a regressor $h : X \times \{1, ..., K\} \to \mathbb{R}$ minimizing weighted squared loss

$$\text{cost}(h, D) = \mathbf{E}_{(x,w,c,i)\sim D}[w(h(x, i) - c)^2].$$

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Let

$$(x, a, r, p) \to (x, 1/p, -r, a)$$

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vw –cb 2 –cb_type mtr rcv1.train.txt.gz -c –ngram 2 –skips 4 -b 24 -l 0.25

Progressive loss: 0.0460

Train method 3: Any RL algorithm

Dipendra prefer Policy Gradient

Summary of methods

1. Deployment. Aka A/B testing. Gold standard for measurement and cost.
2. Direct Method. Often used by people who don't know what they are doing. Some value when used in conjunction with careful exploration.
3. Inverse probability. Unbiased, but possibly high variance.
4. Double robust. Best analyzed offline method? Unbiased + reduced variance.
5. Multitask Regression. Computationally cheapest. Maybe best?

Bibliography: Exploration

Inverse An old technique, not sure where it was first used.

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